Improving the Assimilation of GPS Radio Occultation Observations in the Lower Troposphere

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The National Centers for Environmental Prediction (NCEP) have been assimilating Global Positioning System (GPS) Radio Occultation (RO) observations in its global data assimilation system since 2007. The benefits of incorporating RO observations into the observing system have been demonstrated worldwide, proving that RO data contain very important information on the thermodynamic state of the atmosphere.

Despite the fact that GPS RO technology is a promising tool in recovering the Planetary Boundary Layer (PBL) structure and significant advances have been seen in the GPS RO field during the past decade, there are still some serious issues that affect the assimilation of these observations in the lower moist tropical troposphere.

Indeed, Numerical Weather Prediction (NWP) centers using GPS RO data are rejecting most GPS RO observations at and below the PBL height, significantly limiting the potential benefits of this data type to improve weather forecasting in the lower troposphere.
The reasons for the rejection of these observations are a combination of quality issues that affect retrievals in the lower troposphere and several challenges existing in the data assimilation algorithms.

To address this problem, a methodology is being developed to enable the assimilation of GPS RO data to be extended to the lower moist troposphere.

Limitations associated with so-called super-refraction conditions, which have hitherto prevented NCEP's forward operator NBAM (NCEP’s bending angle method) from applying to the moist lower troposphere, are partially eliminated by a reformulation that has no adverse effects at higher altitudes.

**Super-refraction** refers to vertical gradients of refractivity strong enough to cause the ray curvature to exceed the local geopotential curvature.
The cartoon below shows, schematically, the effect of a super-refraction layer on a bundle of rays artificially arranged in a vertical plane to be instantaneously horizontal as they pass through the vertical midline. Rays above and below the SR layer escape to space in both right and left directions; those starting in the SR layer remain trapped within it.
The more pertinent cartoon for GPS radio occultation is shown below, for the same idealized refractivity profile. Rays enter from space at the left (starting nearly parallel) and almost all return to space – but some spend a VERY long time in the SR layer before eventually escaping – these ones are useless and MUST be excluded (if they are observed at all). But rays penetrating the layer more steeply might be usable.
Super-refraction occurs frequently at the top of low-latitude maritime boundary layers.

It would be really nice to be able to get data within these layers.

While GPS RO rays coming close to tangency with these layers cannot participate in the assimilation, we believe that, in principle, those that penetrate more steeply to greater depths offer valid opportunities for assimilating thermodynamic quantities at these meteorologically important levels.

However, the more erratically distributed errors inherent in bending angle measurements for these deeper-penetrating rays need especially to be analyzed and interpreted carefully in order that an appropriately tuned adaptation of the nonlinear quality control technique can ensure that, through the proper attribution of relative weighting, the assimilation of information from them continues to be acquired in a robust and approximately optimal way.
(Original) 1-dim bending angle function, valid under spherical symmetry of the atmosphere:

\[ \alpha(a) = -2a \int_{a}^{\infty} \frac{d \ln n}{dx} \frac{dx}{(x^2-a^2)^{1/2}} \]  \hspace{1cm} (1)

where \( x \) is the “refractional radius” = \( nr \), \( n(r) \) being refractivity, \( r \) being the radius from the center of symmetry to a point on the ray.

However, under super-refraction conditions, the sequence of \( x \) as a function of \( r \), or of the model vertical level index, becomes non-monotone.

This causes problems when applying the NCEP Bending Angle Method (NBAM)\(^1\), in which we make a “hyperbola” substitution, \( x^2 = a^2 + s^2 \), and integrate w.r.t. \( s \), in order to tame the singularity in the Abel integral at the tangent points, \( x=a \). Therefore rays detected to be contaminated by the SR layer are not used in the present NBAM scheme.

New proposed “NABAM” (NCEP’s Advanced Bending Angle Method)

Assuming \( x \) (refractive radius) is a smooth function of radius \( r \), we can re-express (1) as

\[
\alpha(a) = -2a \int_{r_t}^{\infty} \frac{d \ln n/dr}{(x^2(r) - a^2)^{1/2}} \, dr
\]

(2)

where \( r_t \) is the \( r \) value that verifies \( x(r_t) = a \). Note that this assumption is not valid within the super-refraction layer because there is an infinite number of \( r_i \) that verify \( x(r_i) = a \) (i.e. for all the internal rays). However, observations that have an impact parameter equal to the refractive radius at the super-refraction layer are still rejected form the assimilation algorithms. Thus, we will not be under these circumstances.
Now, the integration grid in (2) is monotone. In order to remove the singularity at the lower limit of the integral \((r=r_t)\) we again apply a hyperbola-type transformation:

\[
r^2 = r_t^2 + s^2
\]

(3)

Under this transformation, (2) becomes

\[
\alpha(a) = -2a \int_0^\infty \frac{d \ln n/ dr}{\sqrt{r^2(x^2(r)-a^2)/s^2}} \; ds = -2a \int_0^\infty \frac{d \ln n/ dr}{\sqrt{r^2(x^2(r)-a^2)/(r^2-r_t^2)}} \; ds
\]

(4)

The tangent point singularity does not really exist when we adopt the new hyperbola transformation at

\[
r_t = \frac{a^2}{n}
\]

(5)

It is replaced by a simple derivative, by l’Hopital’s rule.
The numerical integration of what has now become a smooth integrand, symmetrical about the origin of coordinate $s$, fading exponentially fast towards large $|s|$, can be carried out, as it is in the existing NBAM scheme, by application of the simple and computational cheap trapezoidal rule:

$$\alpha(a) = -a \left[ \beta_0 \Delta s + 2 \sum_{1}^{\infty} \beta_j \Delta s \right]; \quad (6a)$$

with

$$\beta_j = \frac{\frac{d \ln n}{dr} \left[ r^2 (x^2(r) - a^2) / s^2 \right]}{\sqrt{r^2 (x^2(r) - a^2) / s^2}}_j \quad (6b)$$
In addition to these proposed extensions, we are investigating a serially-correlated form of the new “super-logistic” form of the nonlinear quality control.

Briefly, the NLQC methodology prescribes, in a systematic way based on the form of an assumed probability density, how the effective weight attributed to a datum should be down-weighted relative to its nominal “precision”-based weight, as a function (in an implicit way that requires iteration) of the “O-A” for that measurement.

A desirable extension of that idea when the data form a series (such as the consecutive rays in GPS RO) is to formulate the down-weighting function at each specific measurement O, based on an integral transform, or weighted average, of the sequence of the nearby (O-A).

Recursive filters provide one attractive technique for constructing such a formulation. In this way we can account for the fact that gross errors of a series of measurements from the same instrument system are very likely to be highly correlated.
Summary of proposed scientific work

- Improve the assimilation of the observations in the lower troposphere, in particular under super-refraction (SR) conditions (top of the PBL). (We are also planning on implementing variational quality control procedures for RO).
- SR occurs when the gradient of atmospheric refractivity is so large (~ -157 N-units/km) that the ray doesn’t leave the atmosphere.
- Rays that have tangent points inside an elevated atmospheric SR layer are internal (ie. are trapped within the layer).
- Regions of high occurrence frequency of SR are the west coast of major continents in the subtropical oceans and trade wind regions.
- Under SR, the assimilation of GPS RO below the height of the SR layer is an ill-conditioned problem: there is an infinite number of atmospheric states that would reproduce the same exact GPS RO profile.
Summary of proposed scientific work (cont’d)

- When profiles of bending angles are inverted into refractivities at the processing centers, one of the possible solutions is retrieved: the one that has the lowest refractivity value.
- Therefore, refractivity observations are negatively biased under SR conditions at and below the height of the SR layer. In this case, observations need to be rejected in data assimilation.
- On the other hand, observations of bending angle still contain the indetermination - observations might be rejected in a data assimilation system.
- From an observational point of view, we cannot know for sure whether SR occurred (Sergey Sokolovskiy is working on this).
- We must address this issue in the GSI in preparation for the large amount of observations that COSMIC-2 and other GNSS RO missions will bring.