Assimilation of Radar Data in a Convection Permitting NWP System using the Field Alignment Technique

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INTRODUCTION

• DAbyFA, a method proposed by a group of MIT scientists (*Ravela S. et al, 2007*)

• Classical formulations of DA, whether sequential, ensemble-based or variational, are “amplitude adjustment methods”

• Such methods can perform poorly when forecast locations of weather systems are displaced from their observations. Position errors introduce bias

• Characterization of position errors is complex, yet very important for forecasting weather of strong and localized phenomena (tropical cyclones, thunderstorms, squall lines, etc...)

• The issue is not new. For years, “ad-hoc” techniques (“bogussing”) have been used operationally in Tropical Cyclone Forecasting
Other precedents *(references extracted from Ravela S. et al, 2007)*

In the past different objective methods to tackle this problem have been proposed and tested

a) Mariano A.J (1990): contour analysis and melding fields


d) Brewster K.A (2003): another method tested on *storm-scale NWP* with *simulated data*
INTRODUCTION

- Both schemes, 3DVar and EnKF, can perform bad in the presence of position errors (example from Ravela S. et al, 2007)

1-D example built with a 40 members ensemble, perturbed only in amplitude. B-matrix shown down left. “Truth” displaced left about 3*δ, where δ is the width of the “front”. 3DVar analysis and EKF mean analysis appear both distorted. \( \sigma_o \) is substantially less than \( \sigma_b \) (about 1/5). The observation density is 1/10.
The same 1D-example, but with perturbations in position as well. The “truth” is displaced to the left about $3\delta$, where $\delta$ is the perturbation in position. B-matrix is computed from the 40 members ensemble. The distortion in the 3DVar and EKF mean analyses is still important.
INTRODUCTION

Start off from the Bayesian formulation of the DA problem, which gives for the inference of the model state

\[ P(X_n | Y_{0:n}) \propto P(Y_n | X_n) P(X^f_n) \]

The method explicitly represents position errors by introducing in the analysis control space a displacement vector field \( q \), defined in each analysis grid point, that gives the deformation necessary to minimize these position errors.

The inference for the model state now becomes (omitting time indexes)

\[ P(X, q | Y) \propto P(Y | X, q) P(X^f | q) P(q) \]

"Data likelihood". Connects observations to the displaced model state.

The "amplitude prior". Says that the forecast statistics are conditioned on the displacement field \( q \) (e.g. \( B(q) \)).

"Displacement prior", enables the introduction of smoothness constraints on the \( q \) field.
In the usual assumption of gaussian statistics for these component PDFs

a) Data Likelihood \[ P( Y | X, q ) \propto \exp \left( -\frac{1}{2} ( Y - H X (p) )^T R^{-1} ( Y - H X (p) ) \right) \]

where \( X ( p = r - q ) \) represents \( X \) displaced by \( q \)

b) Amplitude prior \[ P( X^f | q ) \propto |B(q)|^{-1/2} \exp \left( -\frac{1}{2}(X(p) - X^f(p))^T B(q)^{-1} (X(p) - X^f(p)) \right) \]

forecast error is Gaussian in the position corrected space

c) Displacement prior \[ P(q) \propto \exp \left( -L(q) \right) \]

\[ L(q) = w_1 / 2 \sum_{j \in \Omega} \text{tr} \left[ \left[ \nabla q_j \right] \left[ \nabla q_j \right]^T \right] + w_2 / 2 \sum_{j \in \Omega} \left[ \text{div} q_j \right]^2 \]

This term expresses the smoothness or “regularization” constraints imposed on the solution for \( q \)
With these definitions of probabilities, the **Field Alignment Cost Function** becomes:

\[
2J_{FA} = X(\vec{p}) - X_f(\vec{p})^T B(\vec{q})^{-1} X(\vec{p}) - X_f(\vec{p}) + Y - H X(\vec{p})^T R^{-1} Y - H X(\vec{p}) + 2L(\vec{q}) - \ln(|B(\vec{q})|)
\]

The solution of this problem is complicated. It is not clear how to compute \(B(\vec{q})\) and the gradients of \(J_{FA}\) are not easy to compute either. Ravela et al. present two ways of overcoming these difficulties by making several approximations.

a) The **one-step algorithm**. An iterative procedure that works with ensembles. The denomination refers to the fact that in this case the minimum is searched simultaneously in amplitude and position.

b) The **sequential solution**. It can be utilized in probabilistic and deterministic approaches alike.
This work is based on the second approach: the "sequential solution" or "two-step algorithm".

Two equations
\[ \frac{\partial J}{\partial X} = 0 \quad (1) \quad ; \quad \frac{\partial J}{\partial q} = 0 \quad (2) \]

Solved sequentially

First: $X$ is fixed to $X^f$ in (2) and then a solution for $q$ is found. This deformation is used to correct the position errors in $X^f$. Second: $X^f(q)$ (the aligned forecast) is used to get an analysis from (1).

Equation (2) is the "alignment equation"

\[ w_1 \Delta \vec{q} + w_2 \nabla \cdot \vec{q} = (\nabla X^f_{\mid p})^T H^T R^{-1} Y - H X^f (\vec{p}) \]

which, due to the dependence of the forcing on $q$, is non-linear and has to be solved iteratively. The forcing term is based on the residual between FG and observations, modulated by the local gradient of the FG. *Indep. of B!*
We easily diagonalize the FA equation by spectral methods, but:

- Boundary conditions? Local operator, forcing term smoothly to zero
- The equation is singular for $k=0$ (mean deformation=0?, No!)

It is found very advantageous to work on an extended domain $2^d$ ($d=2$ here)

Consider a 2D-field $\mathbf{F} = (F_x, F_y)$ such that:

Then:

The $\mathbf{F}$ flux across the internal boundaries = 0

$<\mathbf{F}> /= 0$ in each small box
These symmetry properties translate in the following relations among spectral components

\[
\text{Re} \left[ F_x (k,l) \right] = 0 ; \quad \text{Im} \left[ F_x (k,l) \right] = - \text{Im} \left[ F_x (-k,l) \right] ; \quad \text{Im} \left[ F_x (k,l) \right] = \text{Im} \left[ F_x (k,-l) \right]
\]

\[
\text{Re} \left[ F_y (k,l) \right] = 0 ; \quad \text{Im} \left[ F_y (k,l) \right] = \text{Im} \left[ F_y (-k,l) \right] ; \quad \text{Im} \left[ F_y (k,l) \right] = - \text{Im} \left[ F_y (k,-l) \right]
\]

As it happens, the FA equation preserves these symmetries:

\[
C_x (k,l) Q_x (k,l) + S(k,l) Q_y (k,l) = F_x (k,l)
\]

\[
S(k,l) Q_x (k,l) + C_y (k,l) Q_y (k,l) = F_y (k,l)
\]

\[
C_{x,y} (k,l) , S(k,l) \quad \text{real and}
\]

\[
C_{x,y} (-k,l) = C_{x,y} (k,-l) = C_{x,y} (-k,-l) = C_{x,y} (k,l)
\]

\[
S(-k,l) = S(k,-l) = - S(-k,-l) = - S(k,l)
\]

preserves these symmetries: \( Q_x (k,l), Q_y (k,l) \) also have them

**Corollary:** By giving to the forcing term these characteristics under reflections, we obtain a solution with the desired properties!
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But there are more issues in the implementation of the method than just developing a convenient solver for the FA equation

- **The adaptation to the data source used.** The treatment of the forcing term can be different in each case. In this work we focus on Radar Doppler Wind data generated by several C-band radars of the operational AEMET (Spain) network.

- **The technical issues related to the NWP system employed.** In this work we carry on the prototype development within HARMONIE, a system ensuing from the collaboration between Météo-France and the ALADIN and HIRLAM consortia.
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Calculation of the Obs Operator

\[ w_1 \Delta \ddot{q} + w_2 \nabla \cdot \ddot{q} + (\nabla X^f)^T H^T R^{-1} H X^f - Y = 0 \]

\[ H = H(i, j, \text{lev}, \text{PPI}); \sum_{\text{lev}} H(i, j, \text{lev}, \text{PPI}) = 1; \]

\[ H X = \sum_{\text{lev}} H(i, j, \text{lev}, \text{PPI}) X(i, j, \text{lev}) \]

\[ H^T X = \sum_{\text{PPI}} H(i, j, \text{lev}, \text{PPI}) X(i, j, \text{PPI}) \]
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Treatment of Data Void Areas

Clustering algorithms (e.g. González and Woods, 1992) are utilized to modulate the forcing term.
Other technical issues

• Data quality control
• Scaling of the forcing term
• Smoothing of the forcing term
• Orography features
• Convergence and robustness of the FA process
• etc …
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Blended fc_start 2012092600 ; Wind level 49 FA difference (final-init)
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Encouraging results with the following three-step “hybrid FA+3DVar” scheme

a) Correction of position errors using Field Alignment

b) Upscale and filter the FA corrections using the model error covariances

c) 3DVar assimilation of radar data
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Rationale behind step b)

• Most of the model error is positional: \( \varepsilon_b = \varepsilon_{b \, \text{pos}} + \varepsilon_{b \, \text{other}} \)

• The FA correction is just a correction for this kind of error:

\[
\delta FA = - \varepsilon_{b \, \text{pos}} + \varepsilon_{FA}
\]

• We upscale using a Minimum Variance Unbiased Linear estimate:

\[
\hat{\delta FA}_a = \sum_{\omega \in \Omega} W_{a\omega} \delta FA_{\omega} \quad \text{with} \quad \langle \varepsilon_b \varepsilon_{FA} \rangle = 0
\]

• Which can be approximated by the familiar model error covariances

\[
\begin{align*}
\vec{W}_{a\Omega}^T &= \left( \delta FA_a \delta FA_{\Omega} \right)^T \left( \delta FA_a \delta FA_{\Omega} \right)^{-1} \\
\left( \begin{array}{c}
\varepsilon_{b \, \Omega}^T \\
\varepsilon_{b \, a}^T
\end{array} \right) &+ \left( \begin{array}{cc}
\sigma_{FA}^2 (1) & 0 \\
0 & \sigma_{FA}^2 (\Omega)
\end{array} \right)^{-1}
\end{align*}
\]
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This solution is just the 3D-Var solution in its “incremental formulation”

\[
2J(\delta \mathbf{FA}) = \delta \mathbf{FA}^T \begin{pmatrix} \mathbf{e}_b^T & \mathbf{e}_b \end{pmatrix} \mathbf{M}^{-1} \mathbf{M} \mathbf{e}_b \delta \mathbf{FA} +
\]

\[
(\delta \mathbf{FA}_\Omega - \delta \mathbf{FA})^T \begin{pmatrix} \sigma_{FA}^2(1) & 0 \\ 0 & \sigma_{FA}^2(\Omega) \end{pmatrix}^{-1} (\delta \mathbf{FA}_\Omega - \delta \mathbf{FA})
\]

Therefore the implementation in the current system is done!
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The analysis obtained by this hybrid method contains more small scale information than the standard 3DVar method

More potential for analyses in mesoscale NWP

(Hybrid FA+3DVar)  [Diagram]

Standard 3DVAR (default settings)  [Diagram]
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• Verification of forecasted radial wind using the own radar data:

\[
\text{Error} \equiv < (\text{Fcst} - \text{Radar})^2 >^{1/2}_{\text{PPI}=0.5} + < (\text{Fcst} - \text{Radar})^2 >^{1/2}_{\text{PPI}=1.4}
\]

• Results averaged over more than 150 cases:
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• Case-by-case analysis of the Impact (+3Hours):

![Graphs showing impact over time for different locations: ALMERIA, BARCELONA, MADRID, MURCIA, PALMAMALLORCA, VALENCIA](image-url)
Conclusions

- FA can produce smooth increments at model resolution
- These increments can be easily filtered and extrapolated using statistical interpolation methods
- FA is flow-dependent
- FA is non-linear
- FA is efficient
- No obvious spin-up problems
For the future

• Implement FA also for radar reflectivity data

• Satellite data

• Improve and extent verification with real data

• Consider also studies with synthetic data sources. Model spin-up and model error growth studies

Thank you for your attention!