Data Assimilation for Systems with Many Scales of Motion

Bracco and McWilliams, 2010: *J. Fluid Mech.*

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(with thanks to R. Rotunno)
Assimilation systems now potentially resolve a range of scales, e.g. in atmosphere from synoptic-scale waves to moist convection.

- Revisit Lorenz (1969) model for predictability of homogeneous, isotropic turbulence
- Consider data assimilation based on a uniform grid of observations
- How do estimates of smallest scales change, depending on inherent predictability of the flow?
Begin from equations for 2D non-divergent flow (2DV)

→ linearize error evolution about a reference solution
→ Fourier transform yields evolution eqns for Fourier coeffs
→ evolution eqn for error variance, with expectation over ensemble of errors and of reference states

Assumptions

▷ isotropy and homogeneity, implying solutions depend only $K$ (the wavenumber magnitude)
▷ expectation of 4th moments, when products of quadratics in reference solution with quadratic in error, factors as product of expectation of quadratic products
A Model for Predictability of Turbulence (cont.)

▷ let $z_k$ be error energy, integrated over octave $k$

$$ z_k = \int_{2^{k-1}}^{2^k} e(K) \, dK $$

▷ evolution equation:

$$ \frac{d^2 z_k}{dt^2} = \sum_l C_{kl} z_k $$

▷ reference solution enters only through its $E(K)$, which affects $C_{kl}$
Primer on Energy Spectra

$E(K) = "\text{energy spectrum}\" = \text{energy per unit wave number s.t.}\$

\[
\text{domain averaged energy} = \int E(K) \, dK
\]
Energy Spectra for 2D Turbulence

- $-5/3$ (small-scale forcing) or $-3$ (large-scale) power laws
“White Noise” Spectrum

- If coeffs for all \((k, l)\) are i.i.d., then \(E(K) \sim K\)
Error Growth in 2D Turbulence

- error spectra at $t = 0, 0.1, 0.2, \ldots, 2.0$ for reference flow with $-5/3$ (left) or $-3$ (right) spectrum

Observations and Data Assimilation

Since we know how to evolve the error variance, can now ask how DA works in 2D turbulence. Need to specify obs network and DA procedure.
Observations and Data Assimilation

Observations

- regular network, i.e., $N_o \times N_o$ grid
- observation error $\epsilon \sim N(0, r^2 I)$
- linear transformation of observations $y$:

  $$\text{discrete FT}(y) = \text{Fourier coefficients} + \hat{\epsilon}, \quad \hat{\epsilon} \sim N(0, r^2 N_o^{-2} I)$$
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Data Assimilation

- ignore covariances between wave numbers
- Kalman-filter update for variance at each $K$:

$$e^a(K) = \left(1 - \frac{e^f(K)}{e^f(K) + r^2 N_o^{-2}}\right) e^f(K)$$

- predict $e^f(K)$ at next analysis time, via L69 evolution eqn
- steady-state $e^a(K)$ approached after many cycles
Complete Observations

\[ N_o = 2^{\text{(no. of octaves)}} = 2^{12}, \Delta t = \text{time between obs} = 0.05 \]
Complete Observations (cont.)

$N_o = 2^{\text{no. of octaves}}, \Delta t = 0.01$
Partial Observations

$\Delta N_o = 2^6$, $\Delta t = 0.05$
Summary and Discussion

When can DA recover small scales?

- for $-3$ kinetic-energy spectrum, most rapid error growth occurs at “large eddy” scales and DA recovers small scales.
- for flatter spectra, errors grow fastest at smallest resolved scale
- spectrum of analysis error is peaked at large scales for $-3$ spectrum and at small scales for $-5/3$

Effect is visible only when model resolves a range of scales.
Summary and Discussion (cont.)

Application to atmosphere

▵ KE $\sim K^{-3}$ at large scales; transition to $K^{-5/3}$ at roughly 400 km
▵ suggests two phases of error growth, if sufficient resolution and obs
▵ (models need $dx < 70$ km to see transition, assuming smallest resolved scale of $6dx$)
▵ possibly weak coupling between regimes in atmosphere?

Application to NWP

▵ 60 min is the medium range for convective-scale motions
▵ Time window for validity of TLM will be $\sim$ independent of resolution in $-3$ regime, but will shrink with resolution in $-5/3$ regime
Basic Facts

- 3 spectrum
  - max $\lambda = 5.7$, $e$-folding time = 0.17
  - choose obs frequency $\Delta t = \text{fraction of } \lambda^{-1} = 0.05$

- 5/3 spectrum
  - max $\lambda = 186$, $e$-folding time = 0.0055
  - (max $\lambda$ for truncated system: 117 ($N = 11$), 73 ($N = 10$))
  - choose obs frequency $\Delta t = \text{(fraction of } \lambda^{-1} \text{ for } N=10) = 0.005$