Data Assimilation Experiments using the Back and Forth Nudging and NEMO OGCM

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We present Data Assimilation experiments under the OSSE framework using NEMO ocean model and the Back and Forth Nudging (BFN). This algorithm consists in initially solving the forward equations with a nudging term (forcing term proportional to the difference between the model state and the observations), and then, using the final state as an initial condition, in solving the same equations in a backward direction with a feedback term (with the opposite sign compared to the feedback term of forward nudging). This process is then repeated iteratively until convergence. The implementation of the BFN algorithm has been shown to be very easy, and its convergence very fast, compared to other data assimilation methods \cite{Auroux2008}.

However, several theoretical and numerical studies showed that it was difficult to deal with diffusion processes during backward integrations, leading to instabilities or explosion of the numerical solutions \cite{Auroux2011}. We propose here an improved Back and Forth Nudging algorithm in the context of meteorology and oceanography. In these applications, the theoretical equations are usually diffusive free, or diffusion is small (e.g. Euler’s equation for meteorological processes). But then, in a numerical framework, a diffusive term is often added to the equations (or a diffusive scheme is used), in order to both stabilize the numerical integration of the equations, and take into consideration some subscale phenomena. In such situations, it is physically coherent to change the sign of the diffusion term in the backward integrations, in order to keep the stabilizing role of the diffusion term in both directions in time.

In our experiments on NEMO, two sets of observations are used: either sea-surface height at all grid points and available every day or sea-surface height taken at the satellite (Jason-1) track. We compare the BFN results with results obtained using the NEMOVAR system (4D-Var). Also we tested a "hybrid" method which combines the iterative character of the BFN with a correction step similar to the one used in Kalman Filters (KF). In this case, the resulting algorithm can be seen as a smoother, as long as we use future observations to estimate the initial condition of the system \cite{Cosme2012}.

Our results show that in the case of experiments using sea surface height observations at all grid points, the BFN converges after 10 iterations and performs better than the 4D-Var, when we considered the same computational power for both methods. When we allowed the 4D-Var to do a hundred iterations it performed better in the sense of the mean squared error, but with a cost about one hundred times larger than the BFN cost. Under a more realistic observation network, the performances of both algorithms are close, but the computation cost of BFN is much smaller. The results produced by the hybrid BFN-KF will be compared with the 4D-Var and a smoother.

References